# Methods of gravity field determination: An attempt of an overview

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**Chiostro of the Basilica of S. Pietro in Vincoli**

# Contents

# An apology

- **When invited to this symposium by Mathias Weigelt and Adrian Jäggi, I accepted with pleasure.**
- **For me such presentations are a kind of "personal clearing houses" on interesting scientific issues. I also found that my plan for the presentation would fit rather well into the session.**
- **I will in particular have a closer look at dynamical methods and at acceleration approaches, which became so popular in recent years.**
- **With this plan I more or less leave aside gravity field determination using GOCE accelerometry – which is by no way justified.**
- **I sincerely hope that my presentation will not offend dear friends like *Migliaggio, Reguzzoni, and Sanso*, who started their 2004 JoG article with two remarkable statements:**
  - *For many years, the gravity field of the Earth was only seen by satellite geodesy as the main factor affecting the orbit and consequently it was retrieved together with a number of other orbital perturbations.*
  - *Since the advent of a new generation of accelerometers, non-gravitational perturbations can be separated from the gravity effects and a new era of gravity field estimates from space has been born.*

# Problem Structure

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**The general problem: Many parameters of different type are accessible to satellite geodesy and should be estimated in one step:**

- **Parameters defining the initial state vectors of a satellite arc.**
- **The number of satellites used, the number & lengths of arcs are important solution characteristics.**
- **Dynamical parameters coefficients of the gravity field and scaling factors of other force constituents.**
- **Parameters of observing sites:**
  - **Coordinates of the observing sites in an Earth-fixed system (rigid Earth).**
  - **Coordinates and motion of the sites in a Tissérand system.**
- **Earth rotation and Earth orientation parameters.**
- **Atmosphere parameters defining tropospheric refraction *)**
- **Atmospheric parameters defining ionospheric refraction *)**
- **Technique-specific parameters (biases, etc.)**

***) Technique specific models.**

# Problem Structure

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**Restricted Problem: Gravity field determined with a priori info:**

- **A subset of the parameters previously mentioned are determined in a first step or taken over from an exterior source in the second step focused on gravity field determination.**
- **Example-1: A two-step procedure:**
  - **Step-1: Earth orientation and rotation parameters, the orbit and clock parameters of "auxiliary" satellites (like GNSS satellites) are estimated (or taken over from a trustworthy source) and kept fixed subsequently.**
  - **Step-2: Orbit-specific parameters and dynamical parameters of the satellites used for gravity field determination are determined in the second step.**

**Benefit: Much simpler procedure (in particular when infor-mation is taken over from specialized research entities).**

**Price to be paid: Full consistency of the entire parameter space is lost.**

# Problem Structure



**Restricted Problem: Gravity field determined with a priori info:**

- **Example-2: A three-step procedure:**
  - **Step-1: Earth orientation and rotation parameters, the orbit and clock parameters of "auxiliary" satellites (like GNSS satellites) are estimated (or taken over from a trustworthy source) and kept fixed subsequently.**
  - **Step-2: A kinematic trajectory and the associated covariance matrix are derived with the results of step 1.**
  - **Step-3: Orbit-specific parameters and dynamical parameters of the satellites used for gravity field determination are determined in the third step using the results of step 2.**

***Benefit*: Even simpler procedure (in particular when information is taken over from dedicated research entities).**

***Price*: *Full* consistency of the entire parameter space is lost.**

# From the 20th to the 21st century

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## The 20th century:

- **Observation technique(s): Astrometric, SLR (Satellite Laser Ranging) [and Doppler] observations.**
- **Distribution of observations: Few observations from a sparse tracking network → frequent and at times long observation gaps.**
- **Geodetic satellites: MEOs (Medium Earth Orbiters) at typical heights of 5000-6000 km. Spherical satellites with small A/m-ratios, minimized by construction.**
- **Minimize number of arc-specific parameters by long arcs (days to weeks) and few non-gravitational dynamical parameters thanks to simple models for non-gravitational forces.**
- **Statistical model of the observations: Assumption of independence, unbiased observations (occasional determination of, e.g., time or range biases).**

# From the 20th to the 21st century

| *Satellite* | *A/m* [$m^2$/kg] |
|---|---|
| Moon | $1.3 \cdot 10^{-10}$ |
| LAGEOS | 0.0007 |
| GNSS | 0.02 |
| CHAMP, GRACE | 0.0014*) ; $0.012^{+}$) |
| GOCE | 0.00086*) ; $0.0048^{+}$) |

*) face and roof$^{+}$) values.

- Non-conservative forces, governed by the surface : area ratio A/m, are important in satellite geodesy, in particular for gravity field determination.
- In the 20th century satellites geodetic were made spherical and the A/m ratio as small as possible to render modeling of these forces trivial.
- The geodetic satellites of the 21st century have comparatively large A/m-values.
- The (biased) surface forces are measured by accelerometers.

# From the 20$^{th}$ to the 21$^{st}$ century

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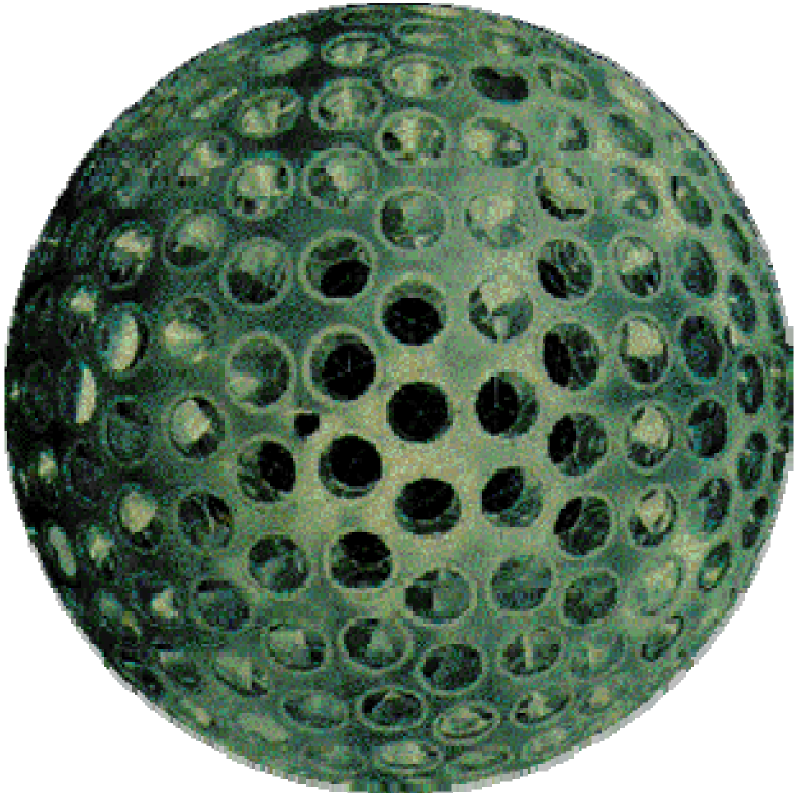
LAGEOS Parameters

| | LAGEOS-1 | LAGEOS-2 |
|---|---|---|
| Sponsor: | United States | United States & Italy |
| Expected Life: | many decades | many decades |
| Primary Applications: | geodesy | geodesy |
| COSPAR ID: | 7603901 | 9207002 |
| SIC Code: | 1155 | 5986 |
| NORAD SSC Code: | 8820 | 22195 |
| Launch Date: | May 4, 1976 | October 22, 1992 |
| RRA Diameter: | 60 cm | 60 cm |
| RRA Shape: | sphere | sphere |
| Reflectors: | 426 corner cubes | 426 corner cubes |
| Orbit: | circular | circular |
| Inclination: | 109.84 degrees | 52.64 degrees |
| Eccentricity: | 0.0045 | 0.0135 |
| Perigee: | 5,860 km | 5,620 km |
| Period: | 225 minutes | 223 minutes |
| Weight: | 406.965 Kg | 405.38 kg |



**Left: LAGEOS-1 und –2 Characteristics**
**Right: „Artist‘s view“ of Lageos-2.**
**The LAGEOS satellites in essence determined the Earth‘s gravity field in the 20th century.**

# From the 20$^{th}$ to the 21$^{st}$ century

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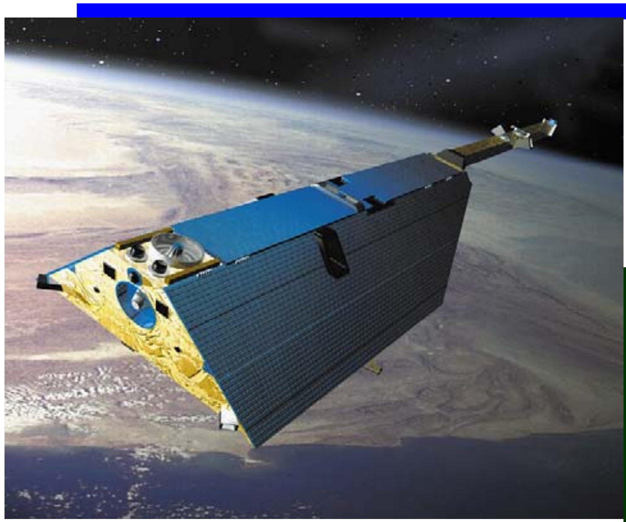
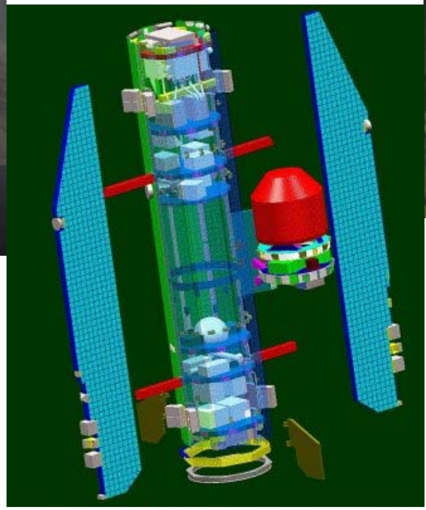
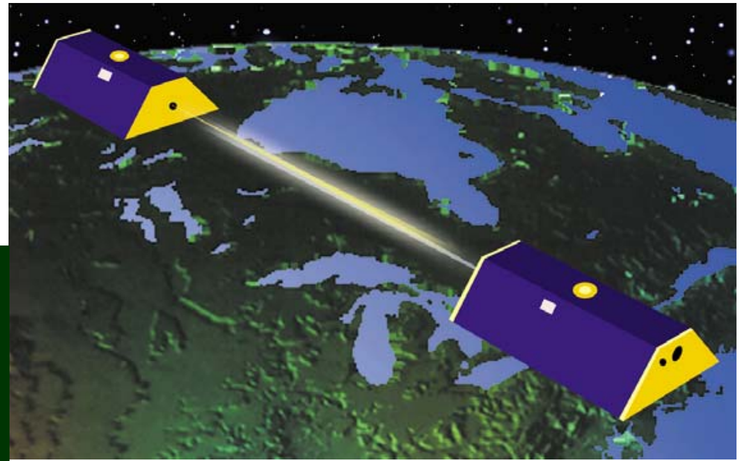


CHAMP

GOCE



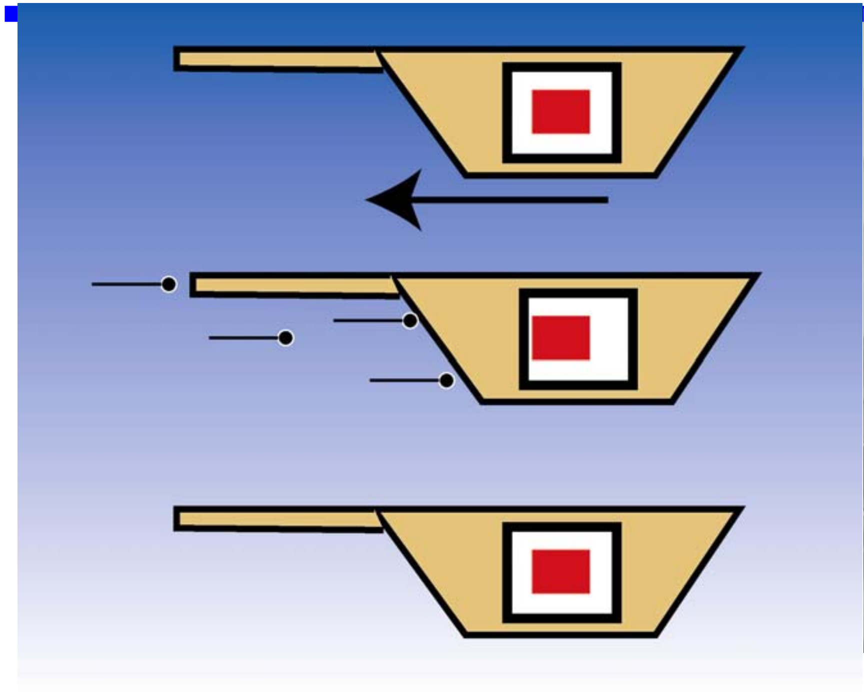


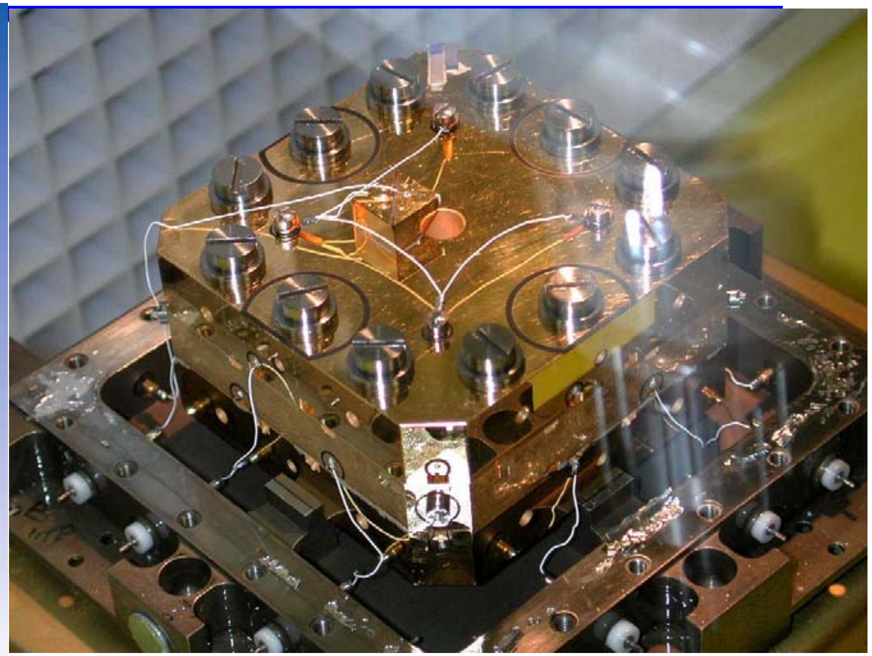
GRACE A and B

**Golden age of gravity field determination was inaugurated with the launch of CHAMP in July 2000 (reentry fall 2010). GRACE was launched 2002 to measure in particular the time-variable part of the gravity field. GOCE was launched on March 17, 2009.**

# From the 20th to the 21st century







**Accelerometers measure the non-gravitational accelerations.**
**Long-term stability of the accelerometers is a tricky issue.**
**→ One reason why the use of long arcs became a problem!**
**→ At least one offset parameter has to be estimated per arc.**

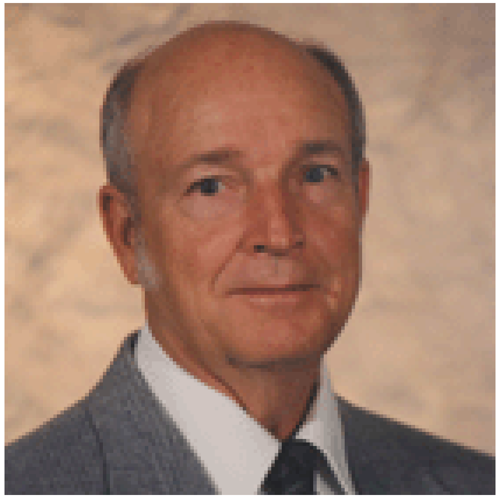
# From the 20th to the 21st century

**The 21st century: Dedicated gravity field missions CHAMP*), GRACE+), and GOCE#).**

- **Uninterrupted observation series with a regular spacing of the GNSS by the on-board receivers (GPS-only for CHAMP and GRACE) are common to all missions.**
- **GRACE in addition measures highly accurate inter-satellite ranges (range rates), GOCE the components of the gravity tensor.**
- **SLR validates the GNSS-derived satellite orbits.**
- **Statistics: Colored noise of all observation types complicates matters.**
- **Geodetic satellites: LEOs (Low Earth Orbiters) at heights below 500km.**
- **Satellites are complex structures with comparatively large A/m-ratios.**
- **Time development of non-conservative forces continuously monitored by 3-d-accelerometers.**
- **It proved to be extremely difficult to represent all orbit-relevant observations (kinematic positions, inter-satellite range (rates), accelerometer measurements) with only few orbit-relevant parameters.**

*) CHAMP = Challenging Mini-satellite Payload, +) GRACE = Gravity Recover And Climate Experiment, #) GOCE = Gravity field and steady-state Ocean Circulation Experiment

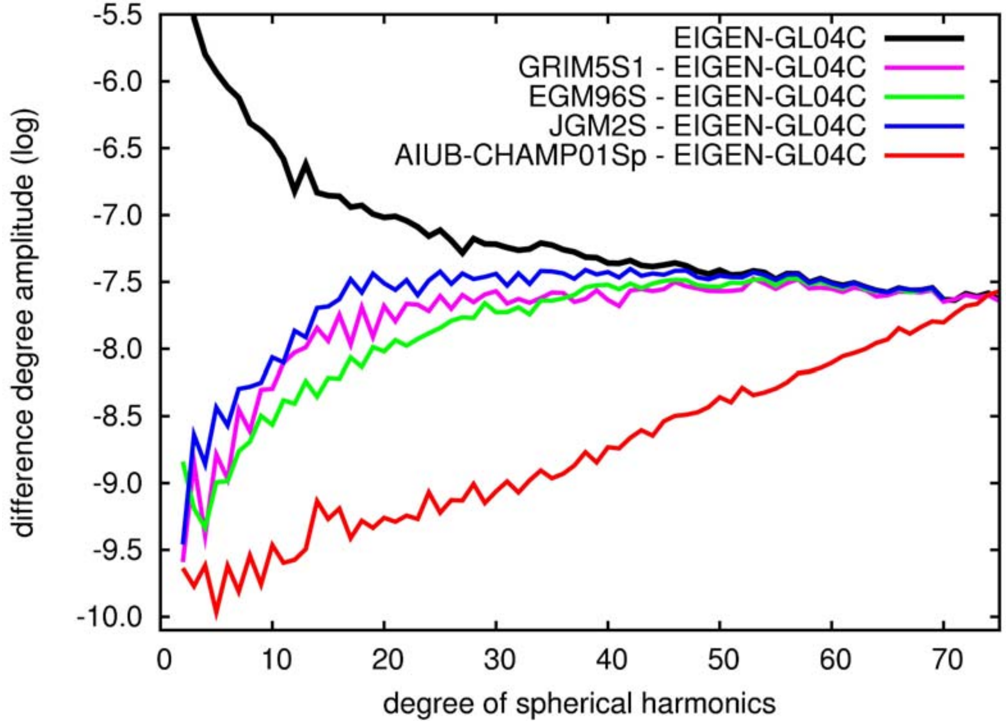
# From the 20th to the 21st century



**The protagonists of the missions:**

- **Christoph Reigber (left)**
- **GRACE: Byron Tapley (center) & Christoph Reigber,**
- **GOCE: Reinhard Rummel (right)**

# From the 20th to the 21st century




Difference degree amplitudes of 20th century gravity fields and of a gravity field based on one year of CHAMP GPS data (red) relative to a 21st century gravity field. → The new missions revolutionize(d) our knowledge of the Earth‘s gravity field.

# Analysis: Dynamical Method

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**Dynamical Method: Each orbit is modeled as a particular solutions of equations of motion within arc (usually one day):**

$$\ddot{r} = f(t, r_0, \dot{r}_0, d_1, d_2, \ldots, d_n, C_{ik}, S_{ik})$$

$$r_0 = r(a, e, i, \Omega, \omega, T_0)$$

$$\dot{r}_0 = \dot{r}(a, e, i, \Omega, \omega, T_0)$$

**Parameters (for the last step of the three-step procedure):**

- **Arc-specific:**
  - **Parameters defining initial state vector, e.g.,** *a, e, …,* $T_0$
  - **Dynamical parameters, e.g., scaling factors of empirical models (constant and/or once-, twice-per rev in user-defined directions, e.g. (*R,S,W*), constant biases of accelerometer measurements).**
- **General:**
  - **e.g., Gravity field parameters** $C_{ik}$, $S_{ik}$.

**Observations: Kinematic positions (with covariance matrices), inter-satellite measurements.**

# Analysis: Short-arc method

The Short-arc method is equivalent to a dynamical method with many more parameters defining the initial state vectors (typically one set per thirty minutes, i.e., of the order of fifty sets per day).

**Advantages**:

- Error propagation is terminated after each arc.
- Initial state vectors absorb many model errors, in particular also errors related to the non-conservative forces, background models.
- Use of accelometer measurements are “superfluous”.

**Disadvantages**:

- Large number of (un)necessary parameters may weaken the determined general parameters: When combining one year of data, there are about 365·300≈100‘000 such parameters.
- Discontinuities of orbit at arc-boundaries, an inconvenience?

# Analysis: CMA



**Ideally, the Celestial Mechanics Approach represents each arc (length typically one day) as a trajectory in the field defined by a simple stochastic differential equation.**

$$\ddot{r} = f(t, r_0, \dot{r}_0, d_1, d_2, ..., d_n, C_{ik}, S_{ik}) + s_{R,t} \cdot e_R + s_{S,t} \cdot e_S + s_{W,t} \cdot e_W$$

$$r_0 = r(a, e, i, \Omega, \omega, T_0)$$

$$\dot{r}_0 = \dot{r}(a, e, i, \Omega, \omega, T_0)$$

- **Only the stochastic properties (expectation values and covariance matrices) of the parameters $s_{R,t}$, $s_{S,t}$, and $s_{W,t}$ are known.**
- **$e_R$, $e_S$, *and* $e_W$ are deterministic unit vectors in radial, along-track and out-of-plane directions.**
- **Solving the above stochastic differential equation includes the estimation of the time series $s_{R,t}$, $s_{S,t}$, and $s_{W,t}$ on top of the deterministic parameters of the conventional method.**
- **The accelerometer measurements are considered as measurements in the parameter estimation process.**

# Analysis: CMA

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**The CMA actually represents each orbital arc as a trajectory in the field of a discretized stochastic differential equation.**

$$t \in [t_i, t_{i+1}]:$$
$$\ddot{r} = f(t, r_0, \dot{r}_0, d_1, d_2, ..., d_n, C_{ik}, S_{ik}) + s_{R,t_i} \cdot e_R + s_{S,t_i} \cdot e_S + s_{W,t_i} \cdot e_W$$

- **The stochastic parameters $s_{R,t_i}$, $s_{S,t_i}$, and $s_{W,t_i}$ are assumed as constant in user-defined intervals $[t_i, t_{i+1}]$ (typical length currently 5-10 minutes).**
- **Only the stochastic properties (in particular expectation values and covariance matrices) of the parameters $s_{R,t_i}$, $s_{S,t_i}$, and $s_{W,t_i}$ are assumed as known (e.g., extracted from the accelerometer time series).**
- **$e_R$, $e_S$, *and* $e_W$ are the unit vectors in radial, along-track and out-of-plane directions.**
- **Parameter estimation is reduced to a conventional least squares process: the parameters $s_{R,t_i}$, $s_{S,t_i}$, and $s_{W,t_i}$ are reduced to normal scaling factors of empirical models.**

# Analysis: Functions of observables

- I learned from Torsten's Ph.D. thesis that the energy balance approach has its roots in the late 1950s. It was applied to satellite geodesy with moderate success in the 2nd half of the 20th success.
- It was probably the first method successfully applied to the new missions, avoiding the explicit solution of the equations of motion.
- Using the measured positions the instantaneous positions and velocities were calculated with (sliding) polynomial approximation / interpolation.
- After getting rid of the non-conservative forces and some tedious algebra, the values of the Earth's potential may be extracted at discrete points in time.
- → The energy balance appoach had an important role in the development of methods.

# Analysis: Functions of observables



**When analyzing an observable, e.g., kinematic positions, one may analyze instead linear or linearized functions of it (e.g., its first or second derivative).**

**All methods reviewed so far are capable of doing that by taking the time derivative of the linearized observation equations (*Δo* arc specific*, Δg* general parameters).**

$$\sum_{i=1}^{n_o} \frac{\partial r_0}{\partial o_i} \cdot \Delta o_i + \sum_{i=1}^{n_g} \frac{\partial r_0}{\partial g_i} \cdot \Delta g_i - \left( r' - r_0 \right) = v; \frac{d^2}{dt^2} \rightarrow$$

$$\boxed{\sum_{i=1}^{n_o} \frac{\partial \ddot{r}_0}{\partial o_i} \cdot \Delta o_i + \sum_{i=1}^{n_g} \frac{\partial \ddot{r}_0}{\partial g_i} \cdot \Delta g_i - \left( \ddot{r}' - \ddot{r}_0 \right) = \ddot{v}}$$

# Analysis: Functions of observables

$$A_o \cdot \Delta o + A_g \cdot \Delta g - (\Delta r) = v$$

$$\ddot{A}_o \cdot \Delta o + \ddot{A}_g \cdot \Delta g - (\Delta \ddot{r}) = \ddot{v}$$

$$\begin{pmatrix} A_o A_o^T & A_o A_g^T \\ (A_o A_g^T)^T & A_g A_g^T \end{pmatrix} \cdot \begin{pmatrix} \Delta o \\ \Delta g \end{pmatrix} = \begin{pmatrix} A_0^T \cdot \Delta r \\ A_g^T \cdot \Delta r \end{pmatrix}$$

$$\begin{pmatrix} \ddot{A}_o \ddot{A}_o^T & \ddot{A}_o \ddot{A}_g^T \\ (\ddot{A}_o \ddot{A}_g^T)^T & \ddot{A}_g \ddot{A}_g^T \end{pmatrix} \cdot \begin{pmatrix} \Delta o \\ \Delta g \end{pmatrix} = \begin{pmatrix} \ddot{A}_0^T \cdot \Delta \ddot{r} \\ \ddot{A}_g^T \cdot \Delta \ddot{r} \end{pmatrix}$$

Observation and normal equations in matrix notation for a particular observable and, e.g., its 2nd time derivative.

# Approximate Solutions

$$\begin{pmatrix} \boxed{A_o A_o^T} & A_o A_g^T \\ (A_o A_g^T)^T & A_g A_g^T \end{pmatrix} \cdot \begin{pmatrix} \boxed{\Delta o} \\ \Delta g \end{pmatrix} = \begin{pmatrix} \boxed{A_0^T \cdot \Delta r} \\ A_g^T \cdot \Delta r \end{pmatrix}$$

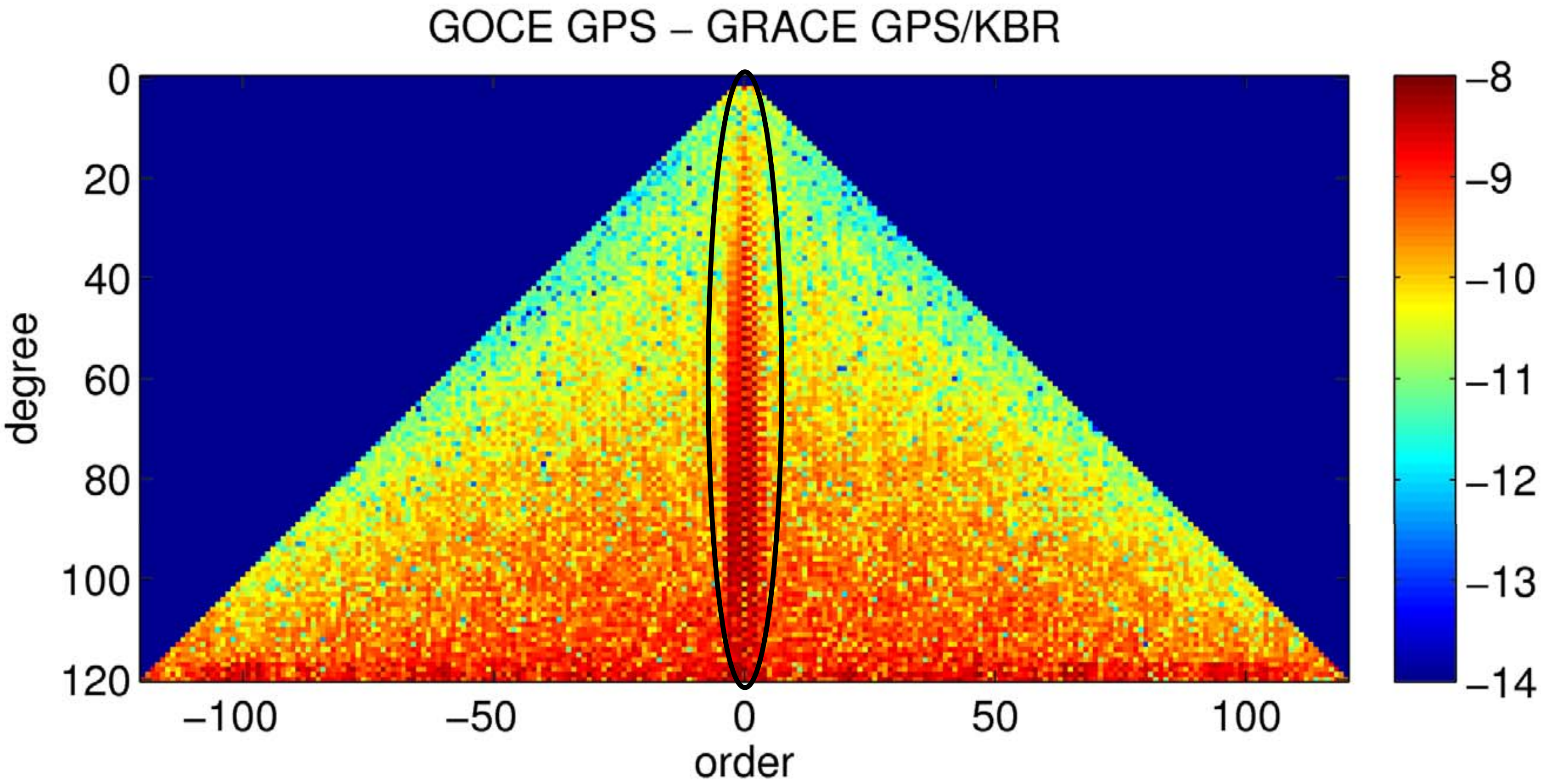
$$\boxed{A_g A_g^T \cdot \Delta g = A_g^T \cdot \Delta r - (A_o A_g^T)^T \cdot \Delta o}$$

$$\begin{pmatrix} \ddot{A}_o \ddot{A}_o^T & \ddot{A}_o \ddot{A}_g^T \\ (\ddot{A}_o \ddot{A}_g^T)^T & \boxed{\ddot{A}_g \ddot{A}_g^T} \end{pmatrix} \cdot \begin{pmatrix} \Delta o \\ \boxed{\Delta g} \end{pmatrix} = \begin{pmatrix} \ddot{A}_0^T \cdot \Delta \ddot{r} \\ \boxed{\ddot{A}_g^T \cdot \Delta \ddot{r}} \end{pmatrix}$$

**Red: Orbit Determination using the original observable in the a priori gravity field, *then* solve for gravity field!**

**Green: Gravity field determination with fixed orbit (e.g. “red one”) → Acceleration approach? Not really, but close (to be continued).**

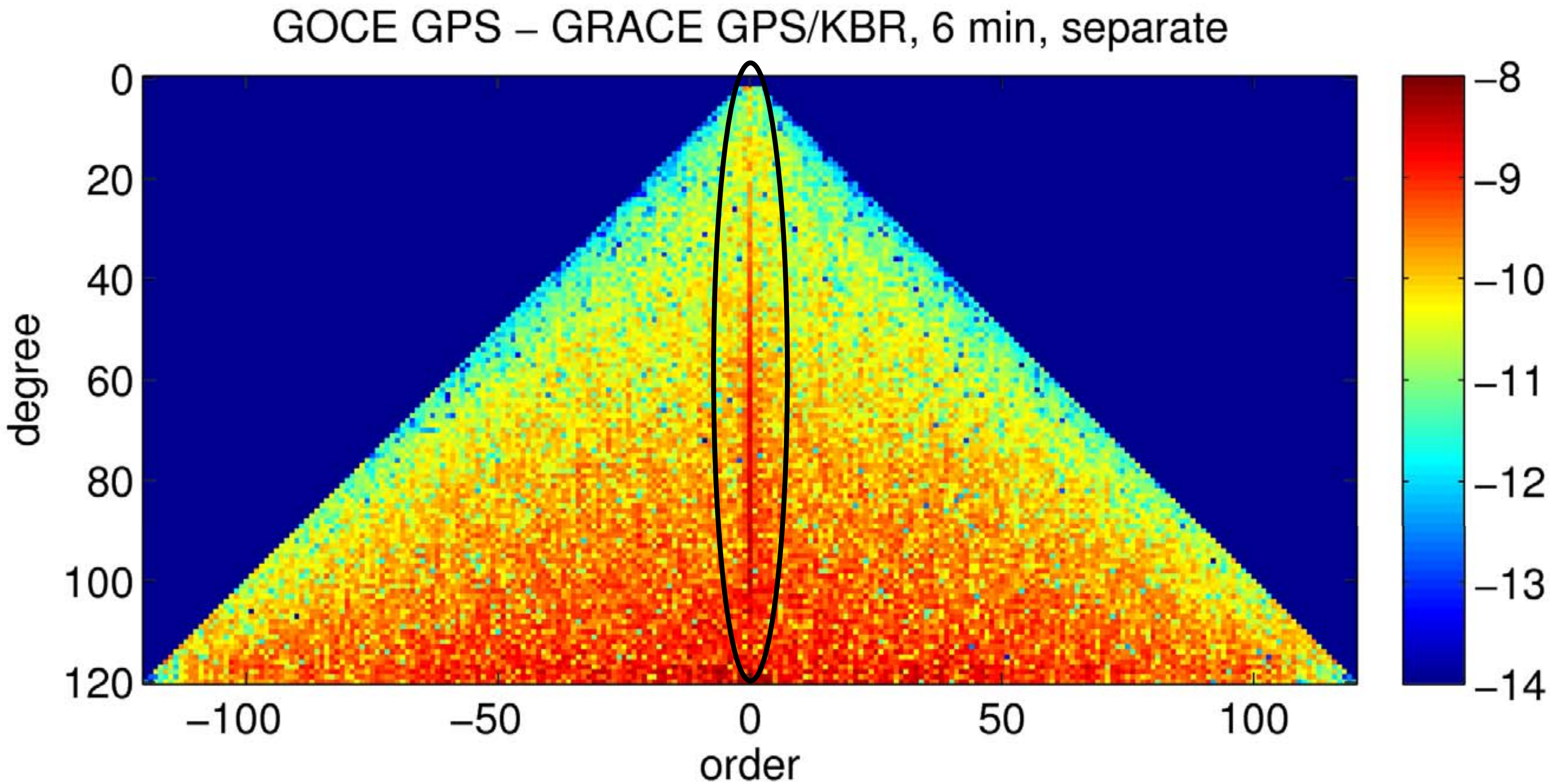
# Approximate Solutions




Correct (en bloc) solution, using kinematic positions (2 months), one set of pulses per 6 minutes.

# Approximate solutions






Approximate ("red") solution, using kinematic positions (2 months), one set of pulses per six minutes. → A priori info taken over!

# Acceleration Approach: Part II

- Dynamical methods and its generalizations are capable of dealing with time derivatives or other functions of the original observables.
- This may even be a necessity – as in the case of the GRACE inter-satellite range-rates.
- From the point of view of dynamical methods it is questionable whether better results could emerge by using linear combinations – in particular when the correlations between the observations are correctly modeled.
- This is why an acceleration approach is not attractive when staying within the dynamical solution pattern!
- Why is the situation different for the acceleration approach?

# Acceleration Approach

$$\boxed{\ddot{r} = f(r, \dot{r}, C_{ik}, S_{ik})}$$

Eqs of motion

$$\frac{\partial}{\partial p} \Rightarrow$$

$$\boxed{\frac{\partial \ddot{r}}{\partial p} = A_0 \cdot \frac{\partial r}{\partial p} + A_1 \cdot \frac{\partial \dot{r}}{\partial p} + \frac{\partial f}{\partial p}}$$

Variational Equations

$$\boxed{\frac{\partial \ddot{r}}{\partial p} = \frac{\partial f}{\partial p}}$$

Partial derivative of acceleration w.r.t. dynamical parameter p

- **Both, Dynamical and acceleration methods, define their functional models by taking the partial derivatives of the equations of motion w.r.t. all parameters p є [$C_{ik}$,$S_{ik}$] of the respective models.**
- **Dynamical methods: → Process leads to the conventional variational equations. Apart from dynamical parameters there are parameters defining the initial state vector, etc.**
- **Acceleration Approach: Dependency on the orbit (and its parameters) is not considered. The observable is thus interpreted as an in situ measurement (at a given position) of the accelerations acting on the satellites.**

# Acceleration vs. Dynamical Approach

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**The acceleration approach and dynamical methods thus fundamentally differ in their goals and products:**

**Dynamical methods**

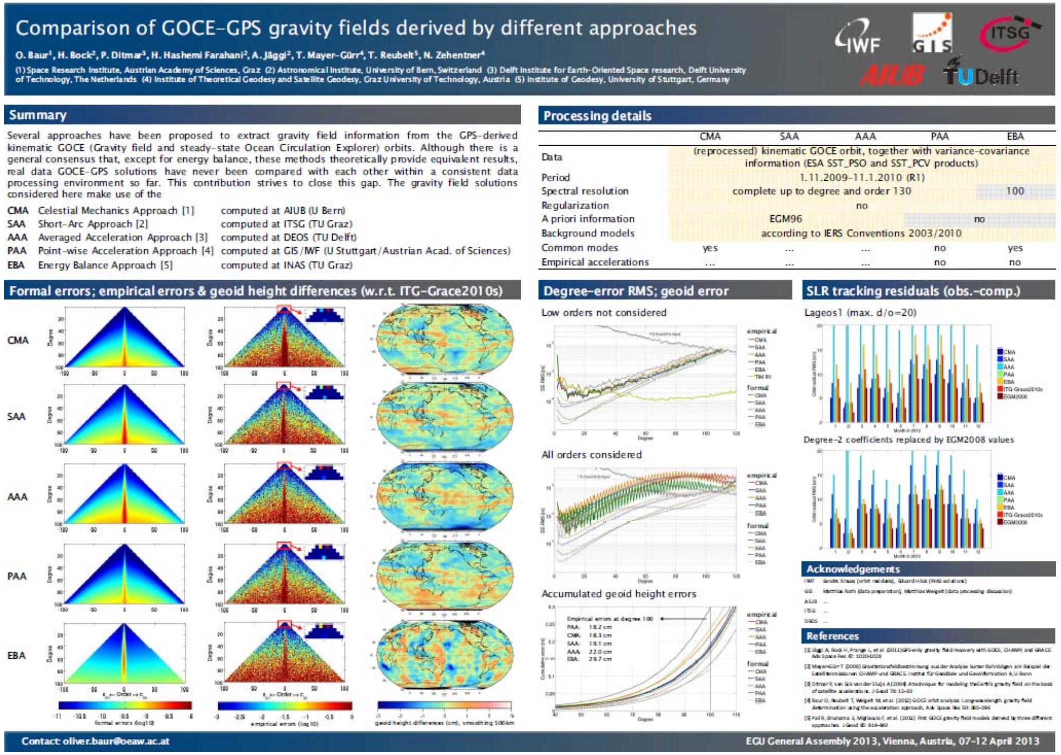
- **generate gravity fields *and* best fitting orbital arcs.**
- **both types of products are consistent:**
  - **the resulting arcs (piecewise) solve the equations of motion within the “new” gravity field**
  - **vice versa, the gravity field best represents all orbits used for its generation.**

**Acceleration approaches**

- **generate gravity fields, *but no orbits*.**
- **The resulting gravity field best represents the accelerations derived from kinematic trajectories.**

**How do gravity fields derived from the two methods compare?**

# Current performance of methods




**All methods mentioned here were compared using kinematic orbits of the GOCE mission (Baur et al., EGU 2013 Vienna).**

**Bottom line: All methods fully exploiting the 3-d accelerations generate comparable results.**

# Summary

- **Comparisons concerning gravity field: All methods making full use of the point-wise accelerations (r.h.s. of the Eqs of motion) generate gravity fields of comparable quality.**
- **Differences of methods:**
  - **Dynamical methods are rather labor intensive, but they are capable of generating gravity fields and consistent orbits.**
  - **Acceleration approach methods generate gravity fields in a rather efficient way.**
- **With the new methods developed since the year 2000 more institutions are able to generate state-of-the-art gravity fields.**
- **This development is mutually stimulating and should be viewed as friendly competition – not as war. → role of IAG?**
- **A priori information should not harm the solutions. It is at times difficult to decide whether this took place. → All methods are sensitive to this problem!**